ERFFECT OF VARIATION OF VISCOSITY, VISCOUS DISSIPATION AND CORIOLIS FORCE ON OBERBECK MAGNETOCONVECTION IN A CHIRAL FLUID IN THE PRESENCE OF A TRANSVERSE MAGNETIC FIELD

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ABSTRACT

The flow and heat transfer characteristics of Oberbeck convection of a chiral fluid in the presence of the transverse magnetic field, coriolis force, viscous dissipation and variation of viscosity with temperature are investigated both analytically and numerically. The coupled non-linear ordinary differential equations governing the flow and heat transfer of the problem are solved analytically using a regular perturbation method valid for small values of Buoyancy parameter N and variable viscosity coefficient R_1. The analytical results are compared with the numerical results obtained using finite difference technique and we found good agreement. The role of temperature dependent viscosity, coriolis force and viscous dissipation on velocity, temperature, skin friction and heat transfer are determined. The results are depicted graphically. From these graphs we found that the velocity is parabolic in nature and increases with an increase in magnetochiral number, M. Physically this is attributed to the fact that magnetochiral number, M introduces small scale turbulences.

Keywords: Chiral Fluids, Oberbeck Convection, Regular Perturbation.

1. INTRODUCTION

In recent years, considerable interest has been evinced in the study of convective heat transfer in a rectangular horizontal or vertical channel or a tube, because of their importance in many applications like in bio-medical engineering, environmental pollution, material science processing and industrial applications like cooling of electronic devices and so on [1]. The available works have been mainly concerned with the study of single, two or multi phases in an ordinary Newtonian fluid with complex flows. These involve non-linear constitutive equations where the interaction between them plays a vital role in controlling the transport processes such as heat and mass transfer and reaction kinetics. A quantitative study of fluid flow using a proper theory is essential to understand the physics of the complex flows. The natural convection is mainly concerned with basic quiescent flow, due to the variation of density with temperature by heating from below and cooling from above, where the temperature difference is maintained parallel to gravity. It is known that, this convection can be controlled by the external constraints like coriolis force due to rotating fluid, and/or by the Lorentz force in an electrically conducting fluid in the presence of a magnetic field. Movement of fluid in a vertical channel, called Oberbeck convection, has also become a specialized topic for research because of its importance in many engineering, industrial and biomechanical applications. Although the Oberbeck convection is investigated in the literature [2], but it pertains mainly to ordinary fluid with uniform viscosity. Convective flows in channels driven by temperature differences of boundary walls have been studied and reported extensively in the literature. On account of their varied importance, such flows have been studied by Raptis A and Perdikis [3]. Hamadah, and Writz [4] have investigated the convective ordinary flow in a vertical channel.

Magnetohydrodynamics convection with heat transfer in a rotating medium has been studied due to its importance in the design of MHD genitors and accelerators in geophysics, nuclear power reactors, soil sciences, astrophysics and so on. Considering this aspect of rotational flows, model studies were carried out on MHD free convection flows in a rotating system by many investigators due to its diverse applications.

The problems occurring in engineering, bio-mechanical and industrial problems, particularly in drug industries and the devices involving fluid lubrication phenomena in a vertical channel involve variation of viscosity with temperature. The results obtained from the flow of fluids with constant viscosity, particularly at high temperature, are very restrictive. Therefore, to predict accurately the flow behavior in such practical problems mentioned above, it is necessary to take into account the variation of viscosity with temperature [5].

In recent years, a number of studies have been done on the fluid phenomenon in the earth involving coriolis force due to rotation to a greater or lesser extent. Fluid flows in rotating channels have been studied by many investigators. Singh et.al [6] have studied the unsteady free convective flow through a rotating porous medium bounded by an infinite vertical porous plate. The convective flows in vertical slots were discussed [7]. The literature is very sparse on convective flows in chiral fluids. In this paper our aim is therefore to study the effect of variation of viscosity on Oberbeck convection in chiral fluid with viscous dissipation and rotation.

A chiral material characterized by either left handed or right handed, is a type of molecule that lacks an internal plane of symmetry and has a non-super-imposable with its mirror image by any amount of rotation and translation [8],[9].We note that the cause for chirality in molecules is the presence of an asymmetric carbon atom. The term chiral in general is used to describe an object that is non-superposable on its mirror image. Human palms are the most universally recognized example of chirality because the left or right palm is non-superposable with its mirror image. A mathematical approach in chirality is the concept of "handedness" either left handed or right handed.

Although the origin of chirality in life is still obscure, it is the source of diverse phenomena at the macromolecular and molecular level, governing our environment and the existence of living organisms. Considerable amount of work has been done during the last three decades on scattering from chiral objects [10],[11]. More recently, Rudraiah et.al have [9] studied the effect of external constraints of magnetic field and velocity shear on the propagation of internal waves in chiral fluid. The effects of temperature dependant viscosity on the boundary layer flow and heat transfer has been investigated [12]. Mostafa et.al [5] dealt with the problem of the effect of thermal radiation and variable viscosity on unsteady Magneto hydrodynamic free convection heat transfer over an infinite horizontal permeable plate in the presence of viscous dissipation. He has used the similarity solutions to solve the coupled non-linear ordinary differential equations. Recently, Atul Kumar et. al [13] have used Arrhenius model (commonly known as exponential model) to study a fully developed flow through a horizontal channel, wherein variable viscosity is decreasing exponentially with temperature.

In the present study, the objective is to investigate the effects of magnetic field, variation of viscosity of chiral fluid, viscous dissipation, rotation of the channel and suction-injection on the free convective MHD flow of chiral fluid in a rotating vertical channel in the presence of a strong magnetic field of uniform strength is applied along the axis of rotation. The required basic equations along with the Maxwell equations, continuity of charges and the constitutive equations for chirality are discussed in the next section. Details of the non dimensional procedure and parameters are described as well as the equations for skin friction and heat transfer are derived. Analytical solutions of the coupled non-linear momentum and energy equations are obtained using a regular perturbation method.

To know the validity of analytical solution, the numerical solution is obtained using the finite difference scheme with SOR. The velocity, temperature, skin friction and rate of heat transfer are computed and the results obtained are depicted graphically, for different controlling parameters influencing the flow characteristics to reveal the underlying physics.
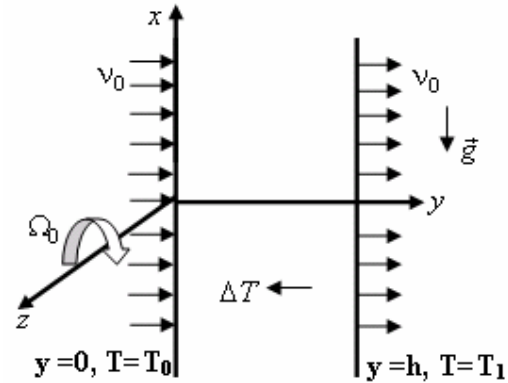
2. MATHEMATICAL FORMULATION

Fig 1. Physical Configuration

We consider an incompressible Boussinesq chiral fluid flowing in a vertical channel bounded by rigid permeable boundaries in the presence of a uniform transverse magnetic field B0 as shown in Figure1.
The basic equations are:
The conservation of mass for an incompressible Boussinesq fluid,

$$\nabla \cdot \vec{q} = 0 \qquad (1)$$

Equation of state for a Boussinesq fluid

$$\rho = \rho_0 \left(1 - \alpha_0 \left(T - T_0\right)\right), \qquad (2)$$

Conservation of momentum

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q} \cdot \nabla\right)\vec{q} + 2\left(\vec{\Omega} \times \vec{q}\right)\right] = -\nabla P + \rho \vec{g}$$
$$+ \nabla \cdot \left(\mu_f \nabla \vec{q}\right) + \vec{J} \times \vec{B}, \qquad (3)$$

Conservation of energy

$$\rho_0 c_p \left[\frac{\partial T}{\partial t} + \left(\vec{q} \cdot \nabla\right)T\right] = k \nabla^2 T + \phi, \qquad (4)$$

$$\phi = 2\mu_f\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \right.$$
$$\left. + + \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)^2\right]$$

Conservation of electric charges in the presence of convective current

$$\vec{J} = \rho_e \vec{q} \qquad (5)$$

where the displacement current $\partial \vec{D}/\partial t$ is neglected compared to convective current $\rho_e \vec{q}$ is

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \left(\rho_e \vec{q}\right) = 0. \qquad (6)$$

The constitutive equations for chiral fluids [9] [10] are,

$$\vec{D} = \varepsilon \vec{E} + i\gamma \vec{B}, \qquad (7)$$

$$\vec{B} = \mu \vec{H} - i\mu\gamma\vec{E}, \qquad (8)$$

The viscosity of chiral fluid is assumed to be temperature dependent and is of the form

$$\mu = \mu_0\left(1 - \alpha_1\left(T - T_0\right)\right) \qquad (9)$$

The flow is assumed to be fully developed, steady and unidirectional parallel to x axis (see Figure1), so that all the physical quantities vary only with respect to y except the pressure, P .The hydrostatic balance is $\partial P/\partial x = \rho_0 g$ .Under these assumptions and making the above equations (1) to (4) dimensionless using the quantities $y^* = y/h$, $u^* = ul\alpha_2 g h^2 \Delta T/\Upsilon$,

$$\rho_e^* = \rho_e /\left(\left|\varepsilon E_0 + i\gamma B_0\right|\right)/h, \quad \theta = T - T_0/\Delta T, \quad \Delta T = T_1 - T_0 \qquad (10)$$

and neglecting the asterisks for simplicity, we get

$$\left(1 - R_1\theta\right)\frac{d^2u}{dy^2} - \left(\mathrm{Re} + R_1\frac{d\theta}{dy}\right)\frac{du}{dy} + \theta + \frac{M\,\mathrm{Re}}{Gr} + \frac{\sqrt{Ta}}{Gr} = 0 \qquad (11)$$

$$\frac{d^2\theta}{dy^2} - Pe\frac{d\theta}{dy} + N\left(1 - R_1\theta\right)\left(\frac{du}{dy}\right)^2 = 0 \qquad (12)$$

where $Gr = B_0\rho_e/\rho_0\nabla Tg\alpha_2$, $N = \rho_0\alpha_2^2 g^2 h^4 \nabla T/\nu_0 K$, $B_0\rho_e\left|\varepsilon E_0 + i\gamma B_0\right|$, $\mathrm{Re} = v_0 h/\Upsilon$, $Pe = v_0 h/\left(K/C_p\rho_0\right)$.

Making the no-slip and isothermal boundary conditions on velocity and temperature dimensionless using equation (10), we get
$$u = 0 \text{ at } y = 0 \text{ , } u = 0 \text{ at } y = 1 \qquad (13)$$
$$\theta = 0 \text{ at } y = 0 \text{ , } \theta = 1 \text{ at } y = 1 \qquad (14)$$

3. ANALYTICAL SOLUTION

Equations (11) and (12) are the coupled non-linear differential equations, whose analytical solutions are obtained using a regular perturbation technique with Buoyancy parameter N as a perturbation parameter. In this technique u and θ are expressed in a series form, given by

$$u = u_0 + Nu_1 + N^2 u_2 + \ldots\ldots\ldots \qquad (15)$$

$$\theta = \theta_0 + N\theta_1 + N^2\theta_2 + \ldots\ldots\ldots \qquad (16)$$

satisfying the boundary conditions (13) and (14).

Substituting equations (15) and (16) into equations (11) and (12) and equating the like powers of N to zero we obtain,

Zeroth order equations:

$$\frac{d^2\theta_0}{dy^2} - Pe\frac{d\theta_0}{dy} = 0 \qquad (17)$$

$$\left(1 - R_1\theta_0\right)\frac{d^2u_0}{dy^2} - \left(\mathrm{Re} + R_1\frac{d\theta_0}{dy}\right)\frac{du_0}{dy} + \theta_0 + \frac{M\mathrm{Re}}{Gr} + \frac{\sqrt{Ta}\,\mathrm{Re}}{Gr} = 0 \quad (18)$$

First order equations:

$$\frac{d^2\theta_1}{dy^2} - Pe\frac{d\theta_1}{dy} + \left(1 - R_1\theta_0\right)\left(\frac{du_0}{dy}\right)^2 = 0 \qquad (19)$$

$$\left(1 - R_1\theta_0\right)\frac{d^2u_1}{dy^2} - \left(\mathrm{Re} + R_1\frac{d\theta_0}{dy}\right)\frac{du_1}{dy} - R_1\theta_1\frac{d^2u_0}{dy^2}$$
$$- R_1\frac{d\theta_1}{dy}\frac{du_0}{dy} + \theta_1 = 0 \qquad (20)$$

The solution of eqn. (16), satisfying the boundary conditions (14), is

$$\theta_0 = \frac{e^{Pey} + 1}{e^{Pe} - 1} \qquad (21)$$

Equations (17) to (19) are differential equations with variable coefficients; it is difficult to find the solutions. To overcome this difficulty, we again use regular perturbation technique with R_1 as a perturbation parameter and u_0 and u_1 are expressed as

$$u_0 = u_{00} + R_1 u_{01}, \qquad (22)$$

$$u_1 = u_{10} + R_1 u_{11}, \qquad (23)$$

with R_1 as a very small parameter.

Eqns. (17) and (19), using eqns. (21) and (22) and equating like power of R_1 to zero, we get

Zeroth order equations are:

$$\frac{d^2u_{00}}{dy^2} - \mathrm{Re}\frac{du_{00}}{dy} - R_1\frac{d\theta_0}{dy}\frac{du_{00}}{dy} + \theta_0 + \frac{M\,\mathrm{Re}}{Gr} + \frac{\sqrt{Ta}\,\mathrm{Re}}{Gr} = 0 \qquad (24)$$

$$\frac{d^2u_{10}}{dy^2} - \mathrm{Re}\frac{du_{10}}{dy} - \mathrm{Re}\frac{d\theta_0}{dy}\frac{du_{10}}{dy} - R_1\frac{d\theta_1}{dy}\frac{du_0}{dy} + \theta_1 = 0 \quad (25)$$

First order equations are:

$$\frac{d^2u_{01}}{dy^2} - \mathrm{Re}\frac{du_{01}}{dy} - \theta_0\frac{d^2u_{00}}{dy^2} - \frac{d\theta_0}{dy}\frac{du_0}{dy} = 0 \qquad (26)$$

$$\frac{d^2u_{11}}{dy^2} - \theta_0\frac{d^2u_{10}}{dy^2} - \theta_1\frac{d^2u_0}{dy^2} - \mathrm{Re}\frac{du_{11}}{dy} - \frac{d\theta_0}{dy}\frac{du_{10}}{dy} - \frac{d\theta_1}{dy}\frac{du_0}{dy} = 0 \qquad (27)$$

In this study, we restrict our solution to the first order approximation. The solutions of the ordinary differential equations (23) to (26) are obtained. Validity of this approximation will be done using numerical technique.

The expressions for u_{00}, u_{01}, u_{10} and u_{11} and the constants involved in it are determined using the boundary conditions. These expressions and constants are very lengthy and hence omitted here but they are included in the computation of the solutions.

3.1 Skin Friction

In many practical applications involving separation of flow, it is advantageous to know the skin friction and heat transfer at the boundaries. These can be determined once we know the velocity and temperature distributions. The skin friction can be calculated from the shear stress τ at the walls which is in the dimensionless form is defined as

$$\tau = \left(1 - R_1\,\theta\,\right)\left(\frac{\partial u}{\partial y}\right) \tag{28}$$

3.2 Heat Transfer

The rate of heat transfer between the fluid and the plate in the dimensionless form is given by

$$Nu = \frac{\partial\theta}{\partial y} \tag{29}$$

3.3 Numerical Solution

To know the accuracy of solutions obtained in section 3 using the approximate method of regular perturbation technique, we find in this section the numerical solution of the coupled nonlinear eqns. (11) and (12) using the second order $O(h^2)$ central finite difference scheme with the following expression.

$$\frac{d\varphi^n}{dy} = \frac{\varphi_{i+1}^n - \varphi_{i-1}^n}{2\Delta y} \tag{30}$$

$$\frac{d^2\varphi^n}{dy^2} = \frac{\varphi_{i+1}^n - 2\varphi_i^n + \varphi_{i-1}^n}{\Delta y^2} \tag{31}$$

where φ^n is the transportive property, $n = 1,2$, $\varphi^1 = u$ and $\varphi^2 = \theta$ .The computational domain is divided into 201 grid points which produce sufficiently fine mesh and generate grid independent solution. Successive over-relaxation technique has been employed for the quick convergence of the solution.

4. RESULTS AND DISCUSSION

The effects of viscosity variation parameter (R_1) on velocity, temperature, skin friction and heat transfer are obtained both analytically and numerically. The analytical solutions are obtained using regular perturbation technique valid for small values of the Buoyancy parameter, $N\,(<1)$ and viscosity variation parameter, $R_1\,(<1)$. The numerical solutions are obtained using Finite difference Technique with Successive Over Relaxation (SOR) method.

The analytical solutions are computed for different values of R_1, Re and M . The results are compared graphically with those obtained from numerical technique. It is observed that the temperature dependent viscosity has a substantial effect on the drag and heat transfer characteristics within the boundary layer. Therefore, it can be concluded that when the viscosity of the chiral fluid is sensitive to temperature variation between the plates, the variable viscosity effect has dominant role in controlling the flow and heat transfer

and causes very little error in the prediction of skin friction and heat transfer rate.

5. CONCLUSIONS

a. An increase in magnetochiral number (M) and suction Reynolds number (Re) increases velocity the and temperature distributions.

b. An increase in viscosity variation parameter (R_1) decreases velocity distribution.

c. The increase in magnetochiral number (M) increase the skin friction and decrease the rate of heat transfer.

d. An increase in Taylor number (Ta) and Grashof number (Gr), there is no significant effect on velocity and temperature profiles.
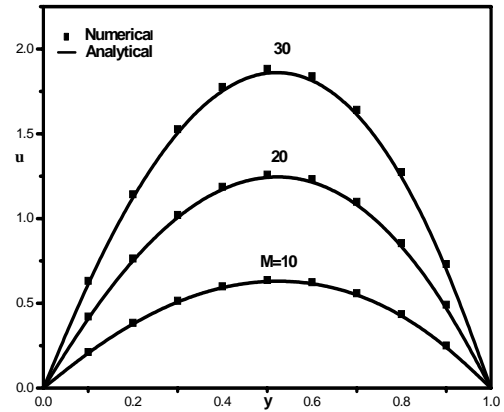

Fig 2: Velocity profiles for different values M when R_1=0.001, N=0.01, Pe=5 and Re=0.

6. FIGURES AND TABLES

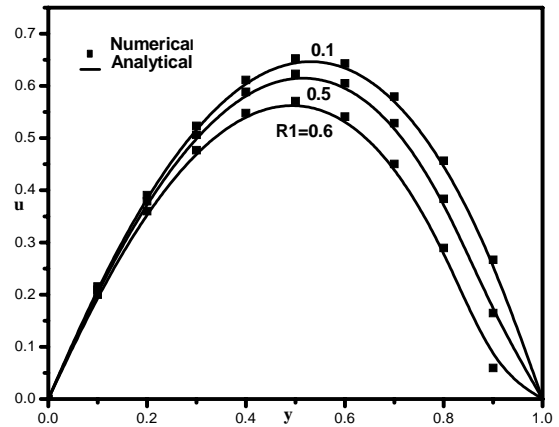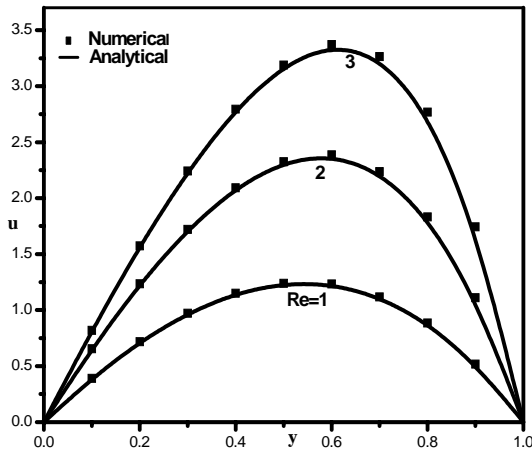Fig 3. Velocity profiles for different values of R_1 when M=10, Re=0.5, Pe=5 and N=0.01.

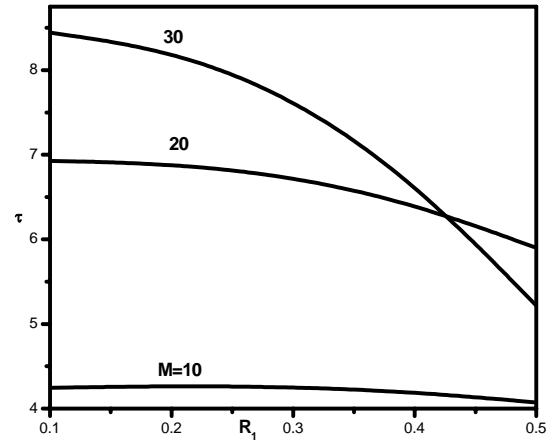Fig 4. Velocity profiles for different values of Re when M=10, Pe=5, R₁=0.001 and N=0.01.

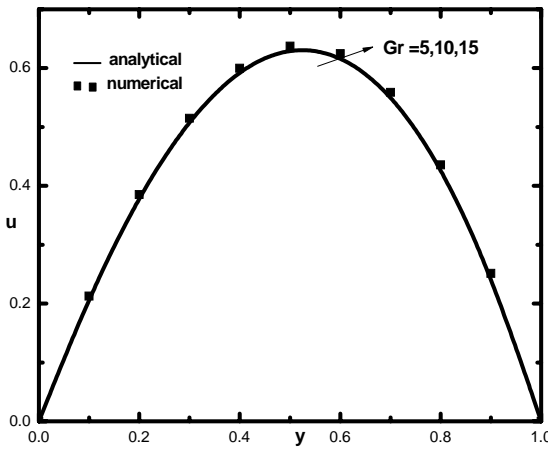Fig 4. Velocity profiles for different values of Re when $M=10$, $Pe=5$, $R_1=0.001$ and $N=0.01$.

Fig 5. Velocity profiles for different values of Re when $M=10$, $Pe=5$, $R1=0.001$, $Ta=5$ and $N=0.01$.

Fig 6. Velocity profiles for different values of Ta when $M=10$, $Pe=5$, $R1=0.001$ and $N=0.01$.

Fig 7. Skin friction v/s R_1 for different values of M when $R1=0.001$, $N=0.1$, $Pe=5$ and $Re=0.5$.
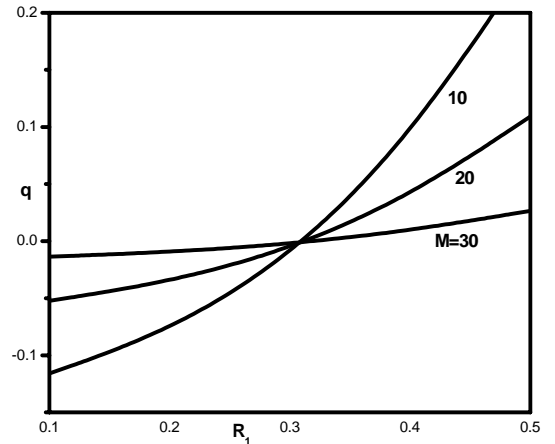

Fig 8. Heat transfer v/s R_1 for different values of M when $Pe=5$, $Ta=5$, $N=0.01$ and $Re=0.5$ at the hotter plate.
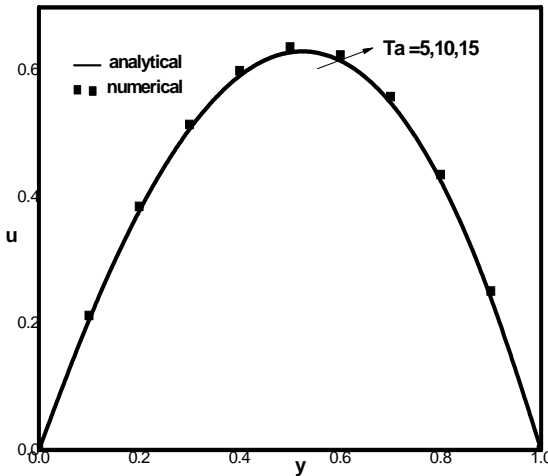
Table.1: Temperature values for different values of magnetochiral number (M) and viscosity variation parameter (R_1)

	Ta=5,Pe=5,Re=0.5 and N=0.01			
	Analytical		**Numerical**	
y	M=10	$R_1=0.5$	M=10	$R_1=0.5$
0	0	0	0	0
0.2	0.01239	0.01452	0.0124	0.0145
0.4	0.04427	0.04699	0.0443	0.047
0.6	0.13053	0.1336	0.1305	0.1336
0.8	0.36479	0.36829	0.3648	0.3683
1.0	1	1	1	1

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ACKNOWLEDGEMENT

8. NOMENCLATURE

Symbol	Meaning	Unit
$\vec{B}$	magnetic field strength	Am^{-1}
$\vec{H}$	magnetic induction	T
$\vec{E}$	electric intensity	V/m
$\vec{D}$	dielectric field	V/m
$\vec{g}$	gravitational field intensity	ms^{-2}
h	width of the channel	m
$\vec{J}$	current density	Am^{-2}
K	thermal conductivity	$W/(Km)$
$\vec{\Omega}$	angular velocity	s^{-1}
Υ	kinematic viscosity	$m^2 s^{-1}$
μ_f	viscosity of the fluid	$Kg.m^{-1}s^{-1}$
μ	magnetic permeability	$Kg.mA^{-2}s^{-2}$
ρ	density of the fluid	$Kg.m^{-3}$
ρ_e	charge density	Cm^{-3}
α_1	coefficient of thermal expansion of viscosity	K^{-1}
α_2	coefficient of thermal expansion of density	K^{-1}
u,v	velocity components	ms^{-1}

Dimensionless numbers

Symbol	Meaning
M	magnetochiral number
Re	suction/injection Reynolds number
Pe	Peclet number
N	Buoyancy parameter
R_1	viscosity variation parameter
γ	chirality coefficient
ε	dielectric constant
ϕ	viscous dissipation
θ	non dimensional temperature
Gr	Grashof number
Ta	Taylor number
τ	skin-friction
Nu	Nusselt number

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